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Resummation of Yukawa enhanced and subleading Sudakov logarithms in longitudinal gauge boson and Higgs production

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Abstract

Future colliders will probe the electroweak theory at energies much larger than the gauge boson masses. Large double (DL) and single (SL) logarithmic virtual electroweak Sudakov corrections lead to significant effects for observable cross sections. Recently, leading and subleading universal corrections for external fermions and transverse gauge boson lines were resummed by employing the infrared evolution equation method. The results were confirmed at the DL level by explicit two loop calculations with the physical Standard Model (SM) fields. Also for longitudinal degrees of freedom the approach was utilized for DL-corrections via the Goldstone boson equivalence theorem. In all cases, the electroweak Sudakov logarithms exponentiate. In this paper we extend the same approach to both Yukawa enhanced as well as subleading Sudakov corrections to longitudinal gauge boson and Higgs production. We use virtual contributions to splitting functions of the appropriate Goldstone bosons in the high energy regime and find that all universal subleading terms exponentiate. The approach is verified by employing a non-Abelian version of Gribov's factorization theorem and by explicit comparison with existing one loop calculations. As a side result, we obtain also all top-Yukawa enhanced subleading logarithms for chiral fermion production at high energies to all orders. In all cases, the size of the subleading contributions at the two loop level is non-negligible in the context of precision measurements at future linear colliders.

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1 Introduction

The high energy behavior of the Standard Model (SM) will become increasingly important at future colliders investigating the origin of electroweak symmetry breaking. At the expected level of precision required to disentangle new physics effects from the SM in the $\mathcal{O}(\leq 1\%)$ regime, higher order *electroweak* radiative corrections cannot be ignored at energies in the TeV-range. The largest contribution is contained in electroweak double logarithms (DL) of the Sudakov type and a comprehensive treatment of those corrections is given in Ref. [1] to all orders. The effects of the mass-gap between the photon and Z-boson has been considered in recent publications [2, 3] since spontaneously broken gauge theories lead to the exchange of massive gauge bosons. In general one expects the SM to be in the unbroken phase at high energies. There are, however, some important differences of the electroweak theory with respect to an unbroken gauge theory. Since the physical cutoff of the massive gauge bosons is the weak scale $M \equiv M_{\rm W} \sim M_{\rm Z} \sim M_{\rm H}$, pure virtual corrections lead to physical cross sections depending on the infrared "cutoff". Only the photon needs to be treated in a semi-inclusive way. Additional complications arise due to the mixing involved to make the mass eigenstates and the fact that at high energies, the longitudinal degrees of freedom are not suppressed. Furthermore, since the asymptotic states are not group singlets, it is expected that fully inclusive cross sections contain Bloch-Nordsieck violating electroweak corrections [4].

It has by now been established that the exponentiation of the electroweak Sudakov DL calculated in Ref. [1] via the infrared evolution equation method [5] with the fields of the unbroken phase is indeed reproduced by explicit two loop calculations with the physical SM fields [2, 3, 6]. One also understands now the origin of previous disagreements. The results of Ref. [7], based on fully inclusive cross sections in the photon, is simply not gauge invariant as already pointed out in Ref. [1]. The factorization used in Ref. [8] is based on QCD and only takes into account contributions from ladder diagrams. In the electroweak theory, the three boson vertices, however, do not simply cancel the corresponding group factors of the crossed ladder diagrams (as is the case in QCD) and thus, infrared singular terms survive for left handed fermions (right handed ones are effectively Abelian) in the calculation of Ref. [8]. The infrared evolution equation method does not encounter any such problems since all contributing diagrams are automatically taken into account by determining the kernel of the equation in the effective regime above and below the weak scale M. It is then possible to calculate corrections in the effective high energy theory in each case yielding the same result as calculations in the physical basis. Thus, the mass gap between the Z-boson and the photon can be included in a natural way with proper matching conditions at the scale M. For longitudinally polarized gauge bosons it was shown in Ref. [9] that the DL kernel can be obtained from the Goldstone boson equivalence theorem.

The picture that emerges has a clear physical interpretation. At high energies,

where particle masses can be neglected, the effective theory is given by an unbroken $SU(2) \times U(1)$ theory for fermions and transversely polarized gauge bosons, and by the equivalence theorem for longitudinally polarized gauge bosons. The contribution from soft photons and collinear terms below the weak scale is determined by QED (including mass terms in the corresponding logarithms). This approach was utilized in Ref. [9] to obtain the subleading (SL) universal terms to all orders for external fermion lines (up to Yukawa enhanced terms) and transversely polarized gauge bosons. Universal, i.e. process independent, are those terms which through Ward identities can be related to external lines and at high energies are given by the contributions to the virtual splitting functions (see Ref. [9]). In addition, there are non-universal angular terms of the type $\log \frac{u}{t} \log \frac{s}{M^2}$ which can be important and should be included at least at one loop. In general, these terms cannot be resummed, they are process dependent and don't factorize with respect to the Born amplitude. At one loop, however, there is a general method for calculating such terms [10] and in practice at most a two loop approach to subleading logarithmic accuracy would be needed.

An important aspect of resumming universal terms is given by the fact that there is a partial cancellation between the DL and SL corrections at energies around a TeV, thus enabling one to see how reliable a DL analysis for a given process really is. In addition, these are predictions of universal terms which can always by used to check higher order calculations which in the electroweak theory are extremely involved due to the number of mass terms and diagrams contributing. It is also conceptually important for a theoretical understanding of the infrared behavior of the SM. By comparing the subleading universal terms with existing one-loop calculations, we gain further evidence for the overall method employed, in particular when it comes to understanding differences between unbroken and broken gauge theories.

From a phenomenological point of view, the corrections to longitudinal gauge boson production are important in case of a strongly interacting $W^\pm$ sector without a fundamental Higgs boson. Our perturbative approach would of course break down in that case, however, it is important to know the precise form of the deviation of the new dynamics from the SM in the TeV range in order to understand the new physics behind the electroweak breaking sector. Corrections to Higgs bosons are important for a precise measurement of the Yukawa couplings at high energies in order to establish the Higgs mechanism. At the level of $6 - 8\%$, these corrections can certainly not be neglected at 1 TeV for determinations of the top-Yukawa coupling at e^+e^- colliders [11].

In this paper, we complete the all orders resummation of all SL universal Sudakov corrections to the SM. While in Ref. [9] have restricted ourselves to calculating terms analogous to QCD, we now consider terms typical for broken gauge theories. These are in particular longitudinal degrees of freedom, processes with external Higgs bosons and Yukawa enhanced logarithmic corrections.

In section 2 we discuss how SL contributions are obtained in the scalar sector and

the application of the equivalence theorem. In section 3 we give results for the virtual contributions to splitting functions involving Goldstone and Higgs bosons fulfilling evolution equations analogous to the Altarelli-Parisi equations. The correctness of this approach to the SL level is verified in section 4 by employing a non-Abelian version of Gribov's bremsstrahlung theorem to processes involving Yukawa enhanced contributions. A similar approach can be utilized to verify analogous corrections for chiral fermions ($b_{\rm L}, t_{\rm L}, t_{\rm R}$) in section 5. Semi-inclusive cross sections for physical observables are given in section 6 and we compare our results with existing one loop calculations in section 7. We discuss the size of the corrections obtained in section 8 and present our conclusions in section 9.

2 The effective high energy theory

In the following we discuss the corrections in a non-Abelian theory with scalar external "quarks", i.e. external scalar bosons charged under the unbroken gauge group. The physical picture is that at high energies, we can use this effective theory via the Goldstone boson equivalence theorem to describe the longitudinal degrees of freedom. The latter will be discussed in detail in section 2.2. An additional complication is given by the presence of Yukawa enhanced logarithmic corrections which are a novel feature of theories with spontaneously broken gauge symmetry. These terms are discussed in section 3. We begin with a discussion of scalar (massless) QCD at high energies.

2.1 Scalar QCD

In this section we are interested in the collinear corrections to external scalars in a non-Abelian gauge theory at the subleading level when the external legs are taken on the mass shell. According to the discussion in Ref. [9] we have to calculate terms contributing to anomalous scaling violations. For purely virtual corrections the invariant matrix element fulfills the following differential equation for massless scalar quarks and all invariants $2p_jp_l \sim s$ large compared to an infrared cutoff μ and denoting $t = \log \frac{s}{\mu^2}$:

$$\left(\frac{\partial}{\partial t} + \beta^{\rm sQCD}\frac{\partial}{\partial g_s} + n_g\left(\Gamma_g(t) - \frac{1}{2}\frac{\alpha_s}{\pi}\beta_0^{\rm sQCD}\right) + n_s\left(\Gamma_s(t) + \frac{1}{2}\gamma_{s\overline{s}}\right)\right) \times\mathcal{M}(p_1, ..., p_n, g_s, \mu^2) = 0 \quad (1)$$

to the order we are working here and where $\mathcal{M}(p_1, ..., p_n, g_s, \mu)$ is taken on the mass-shell. The $\Gamma(t)$ are infrared singular anomalous dimensions leading to DL corrections given in Ref. [9] while the gluon and scalar quark anomalous dimensions describe SL contributions. At higher orders, the subleading RG corrections can be incorporated by including a running coupling in each loop [12]. An additional non-mass suppressed term occurs in the electroweak theory in the case of four scalar scattering amplitudes

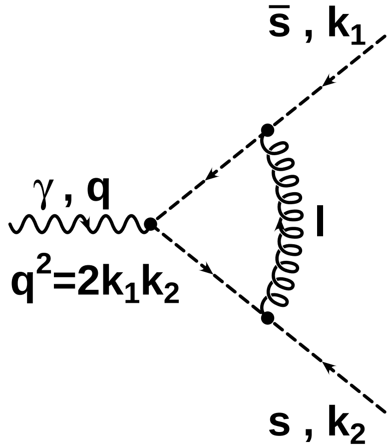

Figure 1: A Feynman diagram determining the DL and SL contributions to scalar quarks in the on-shell scheme. In the massless theory there are scaling violations from loop corrections which can be described by anomalous dimensions.

(such as $\phi^+\phi^- \longrightarrow \phi^+\phi^-$) due to the mass ratios in the coupling ($\lambda \sim \frac{m_{\rm H}^2}{M^2}$) at the Born level. In that case one has to renormalize not only the gauge couplings but also the respective scalar couplings at the one loop level [10]. At higher orders, mass renormalization corrections are subsubleading.

The universality of the corrections follows from the spin independence of the factorization theorems [13] and thus, from the universality of the splitting functions in scalar QCD[1]. Such an equation is of course not physical for a theory with massless gauge bosons, however, in the electroweak theory purely virtual corrections can lead to physical cross sections. Only real soft photon emission needs to be included.

We are interested in determining the scalar quark anomalous dimension $\gamma_{s\overline{s}}$ which gives rise to logarithmic corrections due to scaling violations from the classical case. The factor of $\frac{1}{2}$ in Eq. (1) originates from the fact that it is written for each external leg separately.

Thus we must at one loop calculate the corrections depicted in Fig. 1 with the corresponding Feynman rules for scalar QCD:

$$\mathcal{A}_\nu^{(1)} = -T^b C_F g_s^3 \int \frac{d^n l}{(2\pi)^n} \frac{(-4k_1k_2 + 2l(k_1-k_2) + l^2)(2l + k_1 - k_2)_\nu}{(l^2-\lambda^2+i\varepsilon)((l+k_1)^2+i\varepsilon)((l-k_2)^2+i\varepsilon)} \qquad (2)$$

This is in complete analogy to the situation in QCD [15, 16]. Requiring that the self energy corrections vanish on the mass shell and the on-shell vertex for zero momentum

[1] I would like to thank J. Collins for helpful discussions on this point.

transfer[2] we find the following one loop result:

$$\mathcal{M}^{(1)} = \mathcal{M}^{\rm Born} \left\{ 1 + C_F \frac{g_s^2}{8\pi^2} \left(-\frac{1}{2} \log^2 \frac{s}{\lambda^2} + 2 \log \frac{s}{\lambda^2} - 8 + 2\frac{\pi^2}{3} \right) \right\} \tag{3}$$

where $\mathcal{M}^{\rm Born} = -ig_s T^b (k_1 - k_2)_\nu$. It is important to point out that both leading and subleading logarithmic corrections factorize with respect to the same group factor. The reproduction of this fact in the electroweak theory when compared to exact calculations with the physical fields is crucial in establishing the overall correctness of our approach since we must obtain the factorized form with respect to the effective high energy theory. The finite terms in Eq. (3) are of course irrelevant to the discussion here and the infrared divergent soft and collinear terms were regulated using a fictitious gluon mass term. The difference to the QED result [17] for the on-shell form factor

$$\mathcal{M}^{(1)}_{\rm QED} = \mathcal{M}^{\rm Born}_{\rm QED} \left\{ 1 + \frac{e^2}{8\pi^2} \left(-\frac{1}{2} \log^2 \frac{s}{\lambda^2} + \frac{3}{2} \log \frac{s}{\lambda^2} - 2 + 2\frac{\pi^2}{3} \right) \right\} \tag{4}$$

is (besides the coupling) mainly present in the different collinear divergent subleading term. This term differs due to the different spin of the particle emitting the gauge boson. Here $\mathcal{M}^{\rm Born}_{\rm QED} = -ie \langle k_1, \tau | \gamma_\nu | k_2, \tau \rangle$ as usual and replacing e^2 with $C_F g_s^2$ we obtain the QCD result from Eq. (4). Scaling violations for S-matrix elements can be described by calculating the anomalous dimension of the relevant gauge invariant operators. This is due to the fact that for massless theories there is a one to one correspondence between high and low energy scaling [18, 19]. Thus for the subleading scaling violations only the regions of large loop integration l in Eq. (3) are relevant here (the double logarithms lead to infrared singular anomalous dimensions [20]) and the corresponding anomalous dimension can be read off from the subleading logarithmic term:

$$\gamma_{s\overline{s}} = \frac{\partial}{\partial \log \overline{\mu}} (-\delta_{s\overline{s}} + \delta_s) = -C_F \frac{\alpha}{\pi} \tag{5}$$

where $\delta_{s\overline{s}}$ denotes the counterterm from the diagram depicted in Fig. 1, while δ_s corresponds to the wave function renormalization counterterm of the scalar quark. The sum in Eq. 5 is gauge independent. Since the factorization theorems of Refs. [13] do not depend on the spin of the quark, we can resum the leading and subleading virtual logarithmic corrections by using the Altarelli-Parisi equations. To this end we must formulate the above results in terms of the language of the splitting functions for a massless scalar quark. This will be done in the next section. First, however, we are going to discuss the scalar high energy sector in the Standard Model. In particular, there are additional corrections of the Yukawa type which need to be discussed, for which there is no analogue in unbroken gauge theories.

[2] We can perform this renormalization here since the non-Abelian components don't enter for the scalar quark anomalous dimension. The corresponding counterterm includes automatically the wave function renormalization contribution.

2.2 The equivalence theorem

At high energies, the longitudinal polarization states can be described with the polarization vector

$$e_L^\nu(k) = k^\nu/M + \mathcal{O}(M/E_k) \tag{6}$$

The connection between S-matrix elements and Goldstone bosons is provided by the equivalence theorem [21]. It states that at tree level for S-matrix elements for longitudinal bosons at the high energy limit $M^2/s \longrightarrow 0$ can be expressed through matrix elements involving their associated would be Goldstone bosons. We write schematically in case of a single gauge boson:

$$\mathcal{M}(W_L^\pm, \psi_{\rm phys}) = \mathcal{M}(\phi^\pm, \psi_{\rm phys}) + \mathcal{O}\left(\frac{M_{\rm w}}{\sqrt{s}}\right) \tag{7}$$

$$\mathcal{M}(Z_L, \psi_{\rm phys}) = i\mathcal{M}(\chi, \psi_{\rm phys}) + \mathcal{O}\left(\frac{M_{\rm z}}{\sqrt{s}}\right) \tag{8}$$

The problem with this statement of the equivalence theorem is that it holds only at tree level [22, 23]. For calculations at higher orders, additional terms enter which change Eqs. (7) and (8).

Because of the gauge invariance of the physical theory and the associated BRST invariance, a modified version of Eqs. (7) and (8) can be derived [22] which reads

$$k^\nu \mathcal{M}(W_\nu^\pm(k), \psi_{\rm phys}) = C_{\rm w} M_{\rm w} \mathcal{M}(\phi^\pm(k), \psi_{\rm phys}) + \mathcal{O}\left(\frac{M_{\rm w}}{\sqrt{s}}\right) \tag{9}$$

$$k^\nu \mathcal{M}(Z_\nu(k), \psi_{\rm phys}) = iC_{\rm z} M_{\rm z} \mathcal{M}(\chi(k), \psi_{\rm phys}) + \mathcal{O}\left(\frac{M_{\rm z}}{\sqrt{s}}\right) \tag{10}$$

where the multiplicative factors $C_{\rm w}$ and $C_{\rm z}$ depend only on wave function renormalization constants and mass counterterms. Thus, using the form of the longitudinal polarization vector of Eq. (6) we can write

$$\mathcal{M}(W_L^\pm(k), \psi_{\rm phys}) = C_{\rm w} \mathcal{M}(\phi^\pm(k), \psi_{\rm phys}) + \mathcal{O}\left(\frac{M_{\rm w}}{\sqrt{s}}\right) \tag{11}$$

$$\mathcal{M}(Z_L(k), \psi_{\rm phys}) = iC_{\rm z} \mathcal{M}(\chi(k), \psi_{\rm phys}) + \mathcal{O}\left(\frac{M_{\rm z}}{\sqrt{s}}\right) \tag{12}$$

We see that in principle, there are logarithmic loop corrections to the tree level equivalence theorem. The important point in our approach, however, is that the correction coefficients are not functions of the energy variable s:

$$C_{\rm w} = C_{\rm w}(\overline{\mu}, M, g, g') \ , \ C_{\rm z} = C_{\rm z}(\overline{\mu}, M, g, g') \tag{13}$$

The pictorial form of the Goldstone boson equivalence theorem is depicted in Fig.

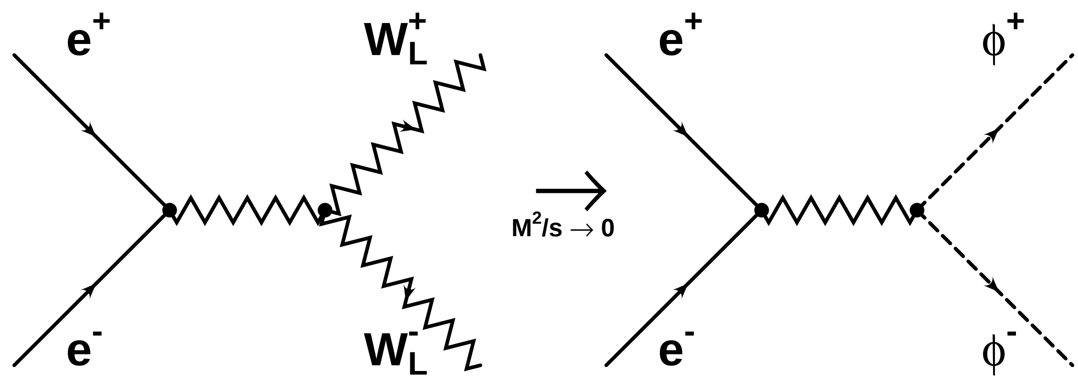

Figure 2: The pictorial Goldstone boson equivalence theorem for W-pair production in e^+e^- collisions. The correct DL-asymptotics for longitudinally polarized bosons are obtained by using the quantum numbers of the charged would be Goldstone scalars at high energies.

2 for longitudinal W-boson production at a linear e^+e^- collider. In the following we denote the logarithmic variable $t \equiv \log \frac{s}{\mu^2}$, where μ is a cutoff on the transverse part of the exchanged virtual momenta k of all involved particles, i.e.

$$\mu^2 \leq \boldsymbol{k}_\perp^2 \equiv \min(2(kp_l)(kp_j)/(p_l p_j)) \tag{14}$$

for all $j \neq l$. The non-renormalization group part of the evolution equation at high energies is given on the invariant matrix element level by [9]:

$$\frac{\partial}{\partial t} \mathcal{M}(L(k), \psi_{\text{phys}}) = K(t) \mathcal{M}(L(k), \psi_{\text{phys}}) \tag{15}$$

and thus, after inserting Eqs. (11), (12) we find that the same evolution equation also holds for $\mathcal{M}(\phi(k), \psi_{\text{phys}})$. The notation here is $L = \{W_L^\pm, Z_L\}$ and $\phi = \{\phi^\pm, \chi\}$, respectively. Thus, the $\log \frac{s}{\mu^2}$ dependence in our approach is unrelated to the corrections to the equivalence theorem, and in general, is unrelated to two point functions in a covariant gauge at high energies where masses can be neglected. This is a consequence of the physical on-shell renormalization scheme where the $\overline{\text{MS}}$ renormalization scale parameter $\overline{\mu} \sim M$. Physically, this result can be understood by interpreting the correction terms C_{w} and C_{z} as corrections required by the gauge invariance of the theory in order to obtain the correct renormalization group asymptotics of the physical Standard Model fields. Thus, their origin is not related to Sudakov corrections. In other words, the results from the previous section should be applicable to the subleading scalar sector in the electroweak theory regarding a non-Abelian scalar gauge theory as the effective description in this range. The only additional complication in the Standard Model is the presence of subleading Yukawa enhanced logarithmic corrections which will be discussed below. It is also worth noticing, that at one loop, the authors

of Ref. [10] obtain the same result for the contributions from the terms of Eq. (13). In their approach, where all mass-singular terms are identified and the renormalization scale $\overline{\mu} = \sqrt{s}$, these terms are canceled by additional corrections from mass and wave function counterterms. At higher orders it is then clear that corrections from two point functions are subsubleading in a covariant gauge.

3 Subleading corrections from splitting functions

In an axial gauge, collinear logarithms are related to corrections on a particular external leg depending on the choice of the four vector n_ν [14]. In a covariant gauge, the sum over all possible insertions shown in Fig. 3 with all invariants large ($\sim s$) is reduced to a sum over all n-external legs due to Ward identities. Overall, these corrections factorize with respect to the Born amplitude. We can therefore adopt the strategy to extract the gauge invariant contribution from the external line corrections on the invariant matrix element at the subleading level. In Ref. [9] we showed that in the high energy regime, subleading logarithmic corrections in massless theories are of collinear or RG origin. This is important since it allows us to use the Altarelli-Parisi approach to calculate the subleading contribution to the evolution kernel of Eq. (15). We are here only concerned with virtual corrections and use the universality of the splitting functions to calculate the subleading terms. For longitudinal degrees of freedom we have shown that to logarithmic accuracy the electroweak theory can be described by scalar Goldstone bosons via the equivalence theorem. Thus at high energies, the effective theory is analogous to scalar QCD[3]. For this purpose we use the virtual gauge boson contributions to the splitting functions $P^V_{\phi^\pm\phi^\pm}(z)$, $P^V_{\chi\chi}(z)$ and $P^V_{\rm HH}(z)$ describing the probability to emit a soft and/or collinear virtual particle with energy fraction z of the original external line four momentum. The infinite momentum frame corresponds to the Sudakov parametrization with lightlike vectors. In general, the splitting functions $P_{\rm BA}$ describe the probability of finding a particle B inside a particle A with fraction z of the longitudinal momentum of A with probability $\mathcal{P}_{\rm BA}$ to first order [15]:

$$d\mathcal{P}_{\rm BA}(z) = \frac{\alpha_s}{2\pi} P_{\rm BA} dt \tag{16}$$

where the variable $t = \log \frac{s}{\mu^2}$ for our purposes. It then follows [15] that

$$d\mathcal{P}_{\rm BA}(z) = \frac{\alpha_s}{2\pi} \frac{z(1-z)}{2} \overline{\sum_{spins}} \frac{|V_{A \longrightarrow B+C}|^2}{\boldsymbol{k}_\perp^2} d\log \boldsymbol{k}_\perp^2 \tag{17}$$

[3]Although Yukawa terms are not present in QCD with scalar quarks, we will show in the next section that at subleading level the Yukawa terms can be treated as an additional term in the Altarelli-Parisi splitting function for Goldstone bosons.

where $V_{A\longrightarrow B+C}$ denotes the elementary vertices and

$$P_{\rm BA}(z)=\frac{z(1-z)}{2}\overline{\sum_{spins}}\frac{|V_{A\longrightarrow B+C}|^2}{\boldsymbol{k}_\perp^2} \tag{18}$$

The upper bound on the integral over $d\boldsymbol{k}_\perp^2$ in Eq. (17) is s and it is thus directly related to dt. Regulating the virtual infrared divergences with the transverse momentum cutoff as described above, we find the virtual contributions to the splitting functions for external Goldstone and Higgs bosons:

$$\begin{aligned}&P^V_{\phi^\pm\phi^\pm}(z)=P^V_{\chi\chi}(z)=P^V_{HH}(z)=\\&\left[\left(T_i(T_i+1)+\tan^2\theta_{\rm w}\left(\frac{Y_i}{2}\right)^2\right)\left(-2\log\frac{s}{\mu^2}+4\right)-\frac{3}{2}\frac{m_t^2}{M^2}\right]\delta(1-z)\end{aligned} \tag{19}$$

The functions can be calculated directly from loop corrections to the elementary processes in analogy to QCD [24, 16, 25] and the logarithmic term corresponds to the leading kernel of Ref. [9]. We introduce virtual distribution functions which include only the effects of loop computations. These fulfill the Altarelli-Parisi equations[4]

$$\frac{\partial\phi(z,t)}{\partial t} = \frac{g^2}{8\pi^2}\int_z^1\frac{dy}{y}\phi(z/y,t)P^V_{\phi\phi}(y) \tag{20}$$

The splitting functions are related by $P_{\phi\phi}=P^R_{\phi\phi}+P^V_{\phi\phi}$, where R denotes the contribution from real boson emission. $P_{\phi\phi}$ is free of logarithmic corrections and positive definite. The renormalizations with respect to the Born amplitude as well as the ones belonging to the next to leading terms at higher orders will be indicated below by writing $\alpha(\overline{\mu}^2)\longrightarrow\alpha(s)$.

Inserting the virtual probability of Eq. (19) into the Eq. (20) we find:

$$\begin{aligned}\phi(1,t) &= \phi_0\exp\left\{-\frac{g^2(s)}{8\pi^2}\left[\left(T_i(T_i+1)+\tan^2\theta_{\rm w}\left(\frac{Y_i}{2}\right)^2\right)\left(\log^2\frac{s}{\mu^2}-4\log\frac{s}{\mu^2}\right)\right.\right.\\&\qquad\left.\left.+\frac{3}{2}\frac{m_t^2}{M^2}\log\frac{s}{\mu^2}\right]\right\}\end{aligned} \tag{21}$$

These functions describe the total contribution for the emission of virtual particles (i.e. $z=1$), with all invariants large compared to the cutoff μ, to the densities $\phi(z,t)$ ($\phi=\{\phi^\pm,\chi,{\rm H}\}$). The normalization is not per line but on the level of the cross section.

[4] Note that off diagonal splitting functions do not contribute to the virtual probabilities to the order we are working here. In fact, for virtual corrections there is no need to introduce off-diagonal terms as the corrections factorize with respect to the Born amplitude. The normalization of Eq. (19) corresponds to calculations in two to two processes on the cross section. The results, properly normalized, are process independent.

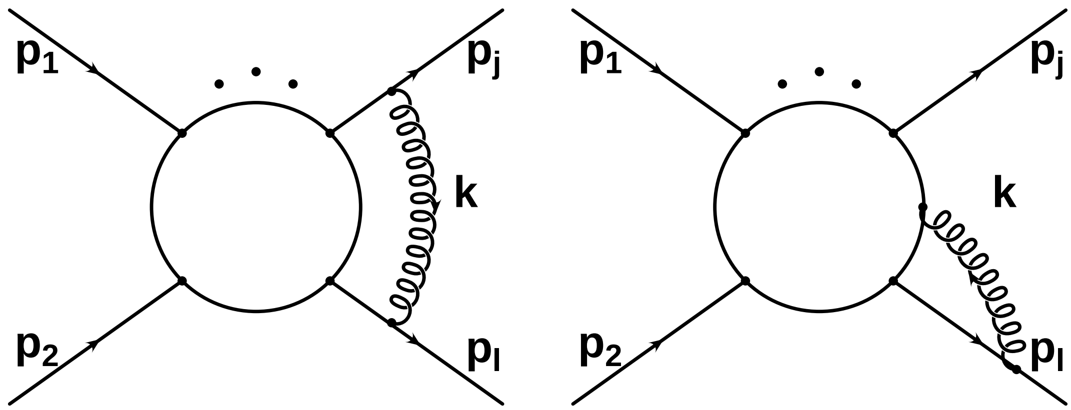

Figure 3: Feynman diagrams contributing to the infrared evolution equation (1) for a process with n external scalar quarks. In a general covariant gauge the virtual gluon with the smallest value of $\boldsymbol{k}_\perp$ is attached to different external lines. The inner scattering amplitude is assumed to be on the mass shell.

For the invariant matrix element involving n_ϕ external scalar particles we thus find at the subleading level:

$$\mathcal{M}(p_1,...,p_n,g_s,\mu^2) = \mathcal{M}(p_1,...,p_n,g_s(s)) \exp\left\{-\frac{1}{2}\sum_{i=1}^{n_\phi} W_i^\phi(s,\mu^2)\right\} \tag{22}$$

where

$$W_i^\phi(s,\mu^2) = \frac{g^2(s)}{16\pi^2}\left[\left(T_i(T_i+1)+\tan^2\theta_{\rm w}\frac{Y_i^2}{4}\right)\left(\log^2\frac{s}{\mu^2}-4\log\frac{s}{\mu^2}\right)+\frac{3}{2}\frac{m_t^2}{M^2}\log\frac{s}{\mu^2}\right] \tag{23}$$

Again we note that the running coupling notation in the Born-amplitude of Eq. (23) denotes the renormalization corrections of the Born amplitude and higher order corrections should be included by inserting a running coupling as in QCD [12]. The functions W_i^ϕ correspond to the probability of emitting a virtual soft and/or collinear gauge boson from the particle ϕ subject to the infrared cutoff μ. Typical diagrams contributing to Eq. (23) in a covariant gauge are depicted in Fig. 3. The universality of the splitting functions is crucial in obtaining the above result.

In addition to the Sudakov corrections in Eq. (23) we also have to include terms corresponding to the renormalization of the mass terms in the Yukawa coupling of the Born amplitude $\left(\sim \frac{m_t^2}{M^2}, \frac{m_H^2}{M^2}\right)$ at the one loop level [10]. At higher orders, mass renormalization terms are connected to two point functions and thus subsubleading.

4 Subleading corrections from Gribov's factorization theorem

In this section we present further evidence for the results of the previous section by employing a non-Abelian version of Gribov's factorization theorem [26]. While in Ref. [26] only real bremsstrahlung corrections are discussed, the form of the virtual soft and collinear divergences must factorize analogously due to the KLN-theorem [27, 28]. The non-Abelian version is discussed in Ref. [1]. The essential point is that to DL accuracy the soft and collinear gauge boson with the smallest $|\boldsymbol{k}_\perp|$ factorizes. From the definition in Eq. (14) it is clear that $\boldsymbol{k}_\perp^2 \approx |k|^2\theta^2$, where θ denotes the angle between the emitted gauge boson of momentum k and the external line emitting this boson, i.e. both soft as well as collinear emission contributions are regularized simultaneously. In the Feynman gauge we then have [1]:

$$
\begin{aligned}
\mathcal{M}(p_1,...,p_n;\mu^2) \;\;=\;\; & \mathcal{M}_{\rm Born}(p_1,...,p_n) - \frac{i}{2}\frac{g_s^2}{(2\pi)^4}\sum_{j,l=1,j\neq l}^{n}\int_{s\gg\boldsymbol{k}_\perp^2\gg\mu^2}\frac{d^4k}{k^2+i\epsilon}\;\frac{p_jp_l}{(kp_j)(kp_l)} \\
& \times T^a(j)T^a(l)\mathcal{M}(p_1,...,p_n;\boldsymbol{k}_\perp^2)\,, \qquad (24)
\end{aligned}
$$

From Eq. (14) it is clear that $\frac{p_jp_l}{(kp_j)(kp_l)} = \frac{2}{\boldsymbol{k}_\perp^2}$ and that Eq. (24) has the required factorized form. For the DL corrections it is convenient to employ the Sudakov parametrization [29] given by:

$$
k = vp_j\; + up_l + k_\perp \qquad (25)
$$

For the boson propagator we use the identity

$$
\frac{i}{k^2+i\varepsilon} = \frac{i}{suv-\boldsymbol{k}_\perp^2+i\varepsilon} = \mathcal{P}\frac{i}{suv-\boldsymbol{k}_\perp^2} + \pi\delta(suv-\boldsymbol{k}_\perp^2) \qquad (26)
$$

writing it in form of the real and imaginary parts (the principle value is indicated by $\mathcal{P}$). The latter does not contribute to the DL asymptotics and at higher orders gives subsubleading contributions. The cutoff will be introduced via the function $\Theta(\boldsymbol{k}_\perp^2-\mu^2)$. Rewriting the measure as $d^4k = d^2k_\perp d^2k_\parallel$ with

$$
\begin{aligned}
d^2k_\perp &= |\boldsymbol{k}_\perp|d|\boldsymbol{k}_\perp|d\varphi = \frac{1}{2}d\boldsymbol{k}_\perp^2 d\varphi = \pi d\boldsymbol{k}_\perp^2 \qquad (27)\\
d^2k_\parallel &= |\partial(k^0,k^x)/\partial(u,v)|dudv = |p_{j_0}p_{l_x} - p_{l_0}p_{j_x}|dudv \approx \frac{s}{2}dudv \qquad (28)
\end{aligned}
$$

where we turn the coordinate system such that the p_j, p_l plane corresponds to $0, x$ and the y, z coordinates to the $k_\perp$ direction so that it is purely spacelike. The last equation follows from $p_i^2 = 0$, i.e. $p_{i_x}^2 \approx p_{i_0}^2$ and

$$
(p_{j_0}p_{l_x} - p_{l_0}p_{j_x})^2 \approx (p_{j_0}p_{l_0} - p_{l_x}p_{j_x})^2 = (p_jp_l)^2 = (s/2)^2 \qquad (29)
$$

Using in addition the conservation of the total group charge

$$\sum_{j=1}^{n} T^a(j)\mathcal{M}(p_1,...,p_j,...,p_n;\boldsymbol{k}_\perp^2) = 0 \tag{30}$$

we arrive at:

$$\begin{aligned}\mathcal{M}(p_1,...,p_n;\mu^2) &= \mathcal{M}_{\rm Born}(p_1,...,p_n) - \frac{2g_s^2}{(4\pi)^2}\sum_{l=1}^{n}\int_{\mu^2}^{s}\frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2}\int_{|\boldsymbol{k}_\perp|/\sqrt{s}}^{1}\frac{dv}{v} \\ &\quad\times C_l\mathcal{M}(p_1,...,p_n;\boldsymbol{k}_\perp^2)\ \ ,\end{aligned} \tag{31}$$

where C_l is the eigenvalue of the Casimir operator $T^a(l)T^a(l)$ (In QCD $C_l = C_A$ for gauge bosons in the adjoint representation of the gauge group $SU(N)$ and $C_l = C_F$ for fermions in the fundamental representation). The differential form of the infrared evolution equation follows immediately from (31):

$$\frac{\partial\mathcal{M}(p_1,...,p_n;\mu^2)}{\partial\log(\mu^2)} = K(\mu^2)\mathcal{M}(p_1,...,p_n;\mu^2)\,, \tag{32}$$

where

$$K(\mu^2) \equiv -\frac{1}{2}\sum_{l=1}^{n}\frac{\partial W_l(s,\mu^2)}{\partial\log(\mu^2)} \tag{33}$$

with

$$W_l(s,\mu^2) = \frac{g_s^2}{(4\pi)^2}C_l\,\log^2\frac{s}{\mu^2}\,. \tag{34}$$

W_l is the probability to emit a soft and almost collinear gauge boson from the particle l, subject to the infrared cut-off μ on the transverse momentum [1]. Note that in distinction to a gluon or photon mass regulator, the cut-off μ does not spoil the gauge invariance of the theory and can take on arbitrary values, i.e. it is not necessarily taken to zero. To logarithmic accuracy, we obtain directly from (34):

$$\frac{\partial W_l(s,\mu^2)}{\partial\log(\mu^2)} = -\frac{g_s^2}{8\pi^2}C_l\log\frac{s}{\mu^2}\,. \tag{35}$$

The infrared evolution equation (32) should be solved with an appropriate initial condition. In the case of large scattering angles, if we choose the cut-off to be the large scale $\sqrt{s}$ then clearly there are no Sudakov corrections. The initial condition is therefore

$$\mathcal{M}(p_1,...,p_n;s) = \mathcal{M}_{\rm Born}(p_1,...,p_n), \tag{36}$$

and the solution of (32) is thus given by the product of the Born amplitude and the Sudakov form factors:

$$\mathcal{M}(p_1,...,p_n;\mu^2) = \mathcal{M}_{\rm Born}(p_1,...,p_n)\exp\left(-\frac{1}{2}\sum_{l=1}^{n}W_l(s,\mu^2)\right) \tag{37}$$

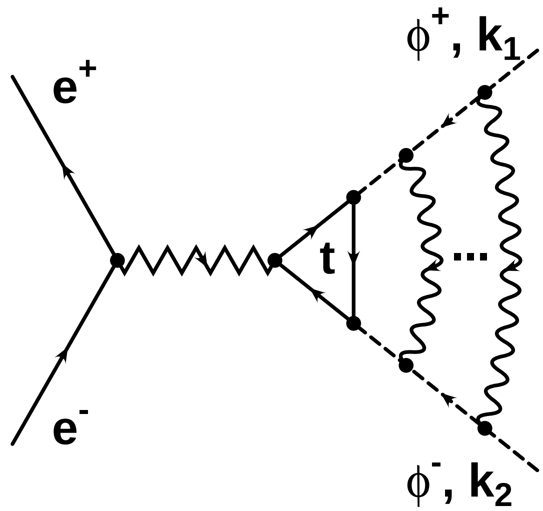

Figure 4: A Feynman diagram yielding Yukawa enhanced logarithmic corrections in the on-shell scheme. At higher orders, the subleading corrections are given in factorized form according to the non-Abelian generalization of Gribov's theorem as described in the text. Corrections from gauge bosons inside the top-loop give only sub-sub leading contributions. DL-corrections at two and higher loop order are given by gauge bosons coupling to (in principle all) external legs as schematically indicated.

This is the exponentiation of Sudakov DL in non-Abelian gauge theories [31]. We now want to apply this result to the electroweak theory for the subleading Yukawa corrections at higher orders. Since we are interested here in corrections to order $\mathcal{O}(\alpha^n L^{2n-1})$, each additional loop correction to the universal subleading terms in the previous section must yield two logarithms, i.e. we are considering DL-corrections to the basic process like the inner fermion loop in Fig. 4. It is of particular importance that all additional gauge bosons must couple to external legs, since otherwise only a subleading term of order $\mathcal{O}(\alpha^n L^{2n-2})$ would be generated. All subleading corrections generated by the exchange of gauge bosons coupling both to external Goldstone bosons and inner fermion lines cancel analogously to a mechanism found in Ref. [30] for terms in heavy quark production in $\gamma\gamma$-collisions in a $J_z = 0$ state. Formally this can be understood by noting that such terms contain an infrared divergent correction. The sum of those terms, however, is given by the Sudakov form factor. Thus any additional terms encountered in intermediate steps of the calculation cancel. For the one loop process in Fig. 4, for instance, we inner include only corrections with top quarks and assume on-shell renormalization of the external Goldstone bosons. Thus the corrections at higher orders factorize with respect to the one loop fermion amplitude and $\mathcal{M}_{\text{“Born''}}(p_1,...,p_n) = \mathcal{M}_{\text{1loop}}(p_1,...,p_n)$. Note that the latter is also independent of the cutoff μ since the fermion mass serves as a natural regulator. In principle we can choose the top-quark mass to be much larger than μ for instance. This freedom is

not present for subleading terms from gauge bosons, such as the angular contributions of the type $\log \frac{u}{t} \log \frac{s}{M^2}$, which furthermore don't factorize with respect to the Born amplitude as mentioned above. Thus, the method suggested in Ref. [32] for the higher order angular terms cannot straightforwardly be justified via the non-Abelian generalization of Gribov's bremsstrahlung theorem[5]. In our case we have for the two loop electroweak DL corrections at the weak scale $\mu = M$:

$$W_l^{\mathrm{ew}}(s, M^2) = \frac{g^2(s)}{16\pi^2}\left[\left(T_i(T_i+1) + \tan^2\theta_{\mathrm{w}} \frac{Y_i^2}{4}\right) \log^2 \frac{s}{M^2}\right] \tag{38}$$

We now want to consider specific processes relevant at future e^+e^- colliders and demonstrate how to apply the non-Abelian version of Gribov's factorization theorem for the higher order corrections. The subleading corrections are then compared to the general splitting function approach of section 3.

In the case of the amplitude of Fig. 4 we must use the quantum numbers of the associated Goldstone bosons and we have the following Born amplitude

$$\mathcal{M}_{\mathrm{Born}}(p_1, ..., p_4) = i \frac{e^2}{2sc_{\mathrm{w}}^2} \langle e_{\mathrm{R}}^- | \gamma^\nu | e_{\mathrm{L}}^+ \rangle (k_1 - k_2)_\nu \tag{39}$$

and at one loop we have two fermion loops contributing (*ttb* and *bbt*). The renormalization condition is provided by the requirement that the corrections vanish at the weak scale, i.e. for $s = M^2$, which amounts to subtracting the vertex for that case. The first diagram of the two is given by

$$\begin{aligned}
\mathcal{A}^{ttb}_{\mathrm{1loop}}(p_1, ..., p_4) &= 3 \sum_{\gamma, Z} \frac{e^4 m_t^2}{2sM^2 s_{\mathrm{w}}^2} \langle e_{\mathrm{R}}^- | \gamma^\nu | e_{\mathrm{L}}^+ \rangle c_+^e \times \\
\int \frac{d^n l}{(2\pi)^n} & \frac{\mathrm{Tr}\left\{ \omega_- \not{l} \omega_+ (\not{l} - \not{k}_2) \gamma_\nu (c_+^t \omega_+ + c_-^t \omega_-)(\not{l} + \not{k}_1) \right\}}{(l^2 - m_b^2 + i\varepsilon)((l+k_1)^2 - m_t^2 + i\varepsilon)((l-k_2)^2 - m_t^2 + i\varepsilon)} + \delta_{\mathrm{ct}}^{ttb} \\
&= \frac{3iQ_t}{16\pi^2 c_{\mathrm{w}}^2} \frac{e^4 m_t^2}{2sM^2 s_{\mathrm{w}}^2} \langle e_{\mathrm{R}}^- | \gamma^\nu | e_{\mathrm{L}}^+ \rangle (B_{23} - B_{23}^M)(k_1 - k_2)_\nu
\end{aligned} \tag{40}$$

where $\omega_\pm = \frac{1}{2}(1 \pm \gamma_5)$ and the chiral couplings are given by $c_\pm^f = Q_f$ for the photon and $c_+^f = \frac{s_{\mathrm{w}}}{c_{\mathrm{w}}} Q_f$ and $c_-^f = \frac{s_{\mathrm{w}}^2 Q_f - T_f^3}{s_{\mathrm{w}} c_{\mathrm{w}}}$ for Z-bosons respectively. The counterterm $\delta_{\mathrm{ct}}^{ttb}$ is chosen such that the logarithmic corrections vanish for $s = M^2$. Thus, the sum of the scalar functions is to logarithmic accuracy $B_{23} - B_{23}^M = -\log \frac{s}{M^2}$. Analogously, we have for the *bbt* quark loop:

$$\mathcal{A}^{bbt}_{\mathrm{1loop}}(p_1, ..., p_4) = 3 \sum_{\gamma, Z} \frac{e^4 m_t^2}{2sM^2 s_{\mathrm{w}}^2} \langle e_{\mathrm{R}}^- | \gamma^\nu | e_{\mathrm{L}}^+ \rangle c_+^e \times$$

[5] Using the $\overline{\mathrm{MS}}$-renormalization scheme, however, the subleading pole structure of QCD scattering amplitudes at the two loop level is determined only by one loop divergences and renormalization group corrections [33].

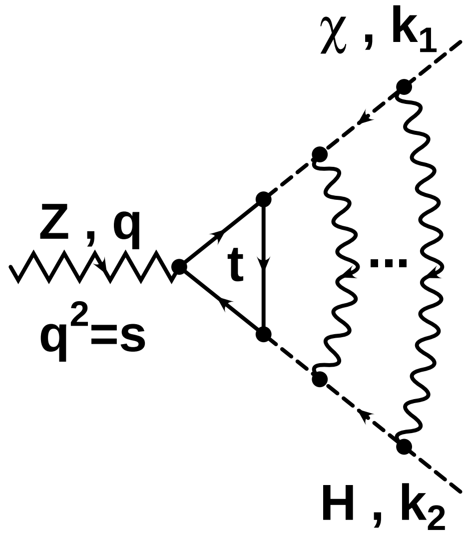

Figure 5: A Feynman diagram yielding Yukawa enhanced logarithmic corrections to external longitudinal Z-bosons and Higgs lines in the on-shell scheme. At higher orders, the subleading corrections are given in factorized form according to the non-Abelian generalization of Gribov's theorem as described in the text. Corrections from gauge bosons inside the top-loop give only sub-sub leading contributions.

$$\int \frac{d^n l}{(2\pi)^n} \frac{\mathrm{Tr}\left\{\omega_+(-\not{l})\omega_-(-\not{l}-\not{k}_1)\gamma_\nu(c_+^b\omega_+ + c_-^b\omega_-)(-\not{l}+\not{k}_2)\right\}}{(l^2-m_t^2+i\varepsilon)((l+k_1)^2-m_b^2+i\varepsilon)((l-k_2)^2-m_b^2+i\varepsilon)} + \delta_{\mathrm{ct}}^{bbt}$$
$$= -\frac{3i(Q_b-T_b^3)}{16\pi^2 c_{\mathrm{w}}^2}\frac{e^4 m_t^2}{2sM^2 s_{\mathrm{w}}^2}\langle e_{\mathrm{R}}^-|\gamma^\nu|e_{\mathrm{L}}^+\rangle(B_{23}-B_{23}^M)(k_1-k_2)_\nu \qquad (41)$$

Adding both results (40) and (41) we find

$$\mathcal{M}_{\mathrm{1loop}}(p_1,...,p_4) = \mathcal{M}_{\mathrm{Born}}(p_1,...,p_4)\left\{1-\frac{g^2}{16\pi^2}\frac{3}{2}\frac{m_t^2}{M^2}\log\frac{s}{M^2}\right\} \qquad (42)$$

and the all orders result to subleading accuracy is given by

$$\mathcal{M}(p_1,...,p_n;\mu^2) = \mathcal{M}_{\mathrm{1loop}}(p_1,...,p_n)\exp\left(-\frac{1}{2}\sum_{l=1}^{n} W_l^{\mathrm{ew}}(s,\mu^2)\right) \qquad (43)$$

The subleading Yukawa corrections from the Altarelli-Parisi in Eq. (22) agree with the corresponding results from the application of the Gribov-theorem in Eq. (43). For longitudinal Z-boson and Higgs production, we note that there is only one non-mass suppressed elementary vertex with two neutral scalars, namely the $Z\chi H$ vertex. As mentioned above, universal terms are related to the massless limit. For the "Born amplitude" of the Higgs-Strahlung vertex we have

$$\mathcal{M}_{\mathrm{Born}}^{\mathrm{Z\chi H}} = \frac{e}{2s_{\mathrm{w}}c_{\mathrm{w}}}(k_1^\nu - k_2^\nu) \qquad (44)$$

The universal Yukawa corrections to both external χ and H states from an off shell Z line are then given by the corrections depicted in the inner fermion loop of Fig. 5. Here we find

$$
\begin{aligned}
&\mathcal{A}^{Z\chi H}_{\rm 1loop}(p_1,...,p_3) = 3\frac{e^3 m_t^2}{4M^2 s_{\rm w}^2} \times \\
&\int \frac{d^n l}{(2\pi)^n} \frac{{\rm Tr}\left\{\gamma_5(\not{l})(\not{l}-\not{k}_2)\gamma^\nu(c^t_+\omega_+ + c^t_-\omega_-)(\not{l}+\not{k}_1)\right\}}{(l^2-m_t^2+i\varepsilon)((l+k_1)^2-m_t^2+i\varepsilon)((l-k_2)^2-m_t^2+i\varepsilon)} + \delta^{Z\chi H}_{\rm ct} \\
&= \frac{6T^3_t}{16\pi^2 s_{\rm w} c_{\rm w}} \frac{e^3 m_t^2}{4M^2 s_{\rm w}^2}(B_{23}-B^M_{23})(k_1^\nu - k_2^\nu) \qquad (45)
\end{aligned}
$$

and thus

$$
\mathcal{M}^{Z\chi H}_{\rm 1loop}(p_1,...,p_3) = \mathcal{M}^{\rm Z\chi H}_{\rm Born}\left\{1 - \frac{3}{2}\frac{e^2 m_t^2}{16\pi^2 s_{\rm w}^2 M^2}\log\frac{s}{M^2}\right\} \qquad (46)
$$

From the same line of reasoning as for the charged Goldstone bosons we find that the all orders result is given by Eq. (43). At the subleading level, this is equivalent to the corresponding corrections obtained in Eq. (22).

5 Top Yukawa corrections for chiral quark production

In Ref. [9] all subleading Sudakov logarithms were resummed assuming that all invariants $2p_jp_l \sim s$. The subleading kernel of the infrared evolution equation was determined by using the virtual contributions to the splitting functions from QCD and applying these results to the high energy regime of the electroweak theory. Soft photon corrections were then added by appropriate matching conditions at the weak scale. We explicitly restricted ourselves in Ref. [9] to the case where all fermions had masses below the weak scale and thus excluded Yukawa enhanced terms. From the arguments of section 4 it is now straightforward to include also top-Yukawa terms for chiral quark final states. These terms occur for left handed bottom as well as top quark external lines. The situation for a typical Drell-Yan process is depicted in Fig. 6 where for the inner scattering amplitude we have two contributions. We neglect all terms of order $\mathcal{O}\left(\frac{m_f^2}{s}, \frac{M^2}{s}\right)$. Using on-shell renormalization we find for the inner amplitude on the left in Fig. 6 for a right handed electron in the initial and a left handed bottom quark in the final state from the $\phi^\pm$ loop for the sum of the γ and Z contributions:

$$
\begin{aligned}
{}^a\mathcal{A}^{\rm DY}_{\rm 1loop} &= -\frac{e^4 m_t^2}{4sM^2 s_{\rm w}^2 c_{\rm w}^2}\langle e^+_{\rm L}|\gamma_\nu|e^-_{\rm R}\rangle \times \\
&\int \frac{d^n l}{(2\pi)^n} \frac{\langle f_{\rm L}|\not{l}(2l-k_1-k_2)^\nu|f_{\rm R}\rangle}{(l^2-m^2_{f'}+i\varepsilon)((l-k_1)^2-M^2+i\varepsilon)((l-k_2)^2-M^2+i\varepsilon)} + {}^a\delta^{\rm DY}_{\rm ct}
\end{aligned}
$$

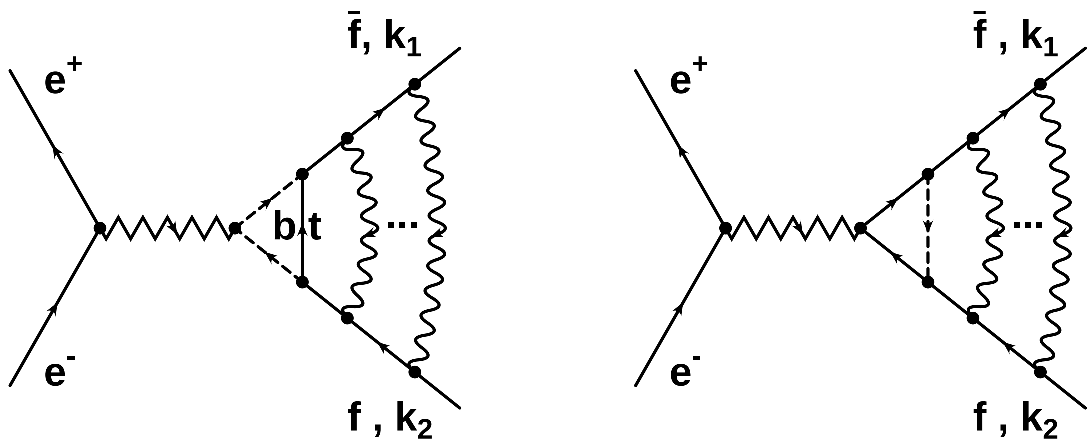

Figure 6: Feynman diagrams yielding Yukawa enhanced logarithmic corrections to the third generation of fermions in the final sate. The inner scattering amplitude is taken on the mass shell. No DL-corrections originate from the inner loop. At higher orders, the subleading corrections are given in factorized form according to the non-Abelian generalization of Gribov's theorem as described in the text. Corrections from gauge bosons inside the Goldstone-boson loop give only sub-sub leading contributions. DL-corrections at two and higher loop order are given by gauge bosons coupling to (in principle all) external legs as schematically indicated.

$$= \; -\frac{i}{32\pi^2}\frac{e^4 m_t^2}{4sM^2 s_{\rm w}^2 c_{\rm w}^2}\langle e^+_{\rm L}|\gamma_\nu|e^-_{\rm R}\rangle\langle f_{\rm L}|\gamma^\nu|f_{\rm R}\rangle(B_{23}-B_{23}^M) \tag{47}$$

The scalar functions at high energy evaluate to $B_{23}-B_{23}^M = -\log\frac{s}{M^2}$ as mentioned above. For the diagram on the right in Fig. 6 we have for the bottom again only the $\phi^\pm$ contribution. Here we find for the sum of the γ and Z contributions:

$$\begin{aligned} {}^b\!\mathcal{A}^{\rm DY}_{\rm 1loop} &= -\frac{e^4 m_t^2 Q_t}{2sM^2 s_{\rm w}^2 c_{\rm w}^2}\langle e^+_{\rm L}|\gamma_\nu|e^-_{\rm R}\rangle \times \\ & \int \frac{d^n l}{(2\pi)^n}\frac{\langle f_{\rm L}|\not{l}\gamma^\nu\not{l}|f_{\rm R}\rangle}{(l^2-M^2+i\varepsilon)((l-k_1)^2-m_t^2+i\varepsilon)((l-k_2)^2-m_t^2+i\varepsilon)} + {}^b\delta^{\rm DY}_{\rm ct} \\ &= \frac{i}{32\pi^2}\frac{e^4 m_t^2 Q_t}{2sM^2 s_{\rm w}^2 c_{\rm w}^2}\langle e^+_{\rm L}|\gamma_\nu|e^-_{\rm R}\rangle\langle f_{\rm L}|\gamma^\nu|f_{\rm R}\rangle(B_{23}-B_{23}^M) \end{aligned} \tag{48}$$

In all cases we renormalize on-shell, i.e. by requiring that the vertex vanishes when the momentum transfer equals the masses of the external on-shell lines. All on-shell self energy contributions don't contribute in this scheme. For external left handed top quarks, the $\phi^\pm$ loop is mass suppressed and we only have to consider the χ and H corrections. They are given by replacing $Q_t \longrightarrow 2Q_t\,(T^3_t)^2$ and $Q_t \longrightarrow \frac{1}{2}Q_t$ in Eq. (48). It turns out that the $Z\chi H$ contributions equal the corrections from the $\gamma\phi^\pm$ and

$Z\phi^\pm$ in the case of the bottom calculation. The Born amplitude is given by:

$$\begin{aligned}
\mathcal{M}^{\rm DY}_{\rm Born} &= i\frac{e^2}{sc_{\rm w}^2}(Q_f - T^3_f)\langle e^+_{\rm L}|\gamma_\nu|e^-_{\rm R}\rangle\langle f_{\rm L}|\gamma^\nu|f_{\rm R}\rangle \\
&= \begin{cases} i\frac{e^2}{6sc_{\rm w}^2}\langle e^+_{\rm L}|\gamma_\nu|e^-_{\rm R}\rangle\langle f_{\rm L}|\gamma^\nu|f_{\rm R}\rangle & , f_{\rm L} = t_{\rm L}, b_{\rm L} \\ i\frac{e^2}{sc_{\rm w}^2}\frac{2}{3}\langle e^+_{\rm L}|\gamma_\nu|e^-_{\rm R}\rangle\langle f_{\rm R}|\gamma^\nu|f_{\rm L}\rangle & , f_{\rm R} = t_{\rm R} \end{cases}
\end{aligned} \tag{49}$$

for top and bottom quarks. In all cases, $\log\frac{M^2}{m_t^2}$ terms can be savely neglected to the accuracy we are working. Thus we find for left handed quarks of the third generation:

$$\mathcal{M}^{\rm DY_L}_{\rm 1loop}(p_1, ..., p_4) = \mathcal{M}^{\rm DY}_{\rm Born}(p_1, ..., p_4)\left\{1 - \frac{g^2}{16\pi^2}\frac{1}{4}\frac{m_t^2}{M^2}\delta_{f,t_{\rm L}/b_{\rm L}}\log\frac{s}{M^2}\right\} \tag{50}$$

For right handed external top quarks we have $\phi^\pm$, χ and H corrections. In that case we observe that the $Z\chi H$, $\gamma\phi^\pm$ and $Z\phi^\pm$ loops have an opposite sign relative to the left handed case. For the corrections corresponding to the topology shown on the right in Fig. 6 we must replace Q_t in Eq. (48) by $Q_f - T^3_f = \frac{1}{6}$ for the $\phi^\pm$ graph. The same contribution is obtained by adding the H and χ loops and we find:

$$\mathcal{M}^{\rm DY_R}_{\rm 1loop}(p_1, ..., p_4) = \mathcal{M}^{\rm DY}_{\rm Born}(p_1, ..., p_4)\left\{1 - \frac{g^2}{16\pi^2}\frac{1}{2}\frac{m_t^2}{M^2}\delta_{f,t_{\rm R}}\log\frac{s}{M^2}\right\} \tag{51}$$

At higher orders we note that the exchange of gauge bosons inside the one loop process is subsubleading and we arrive at the factorized form analogous to the Yukawa corrections in section 4. Since these corrections are of universal nature we can drop the specific reference to the Drell-Yan process and the application of the generalized Gribov-theorem for external fermion lines to all orders yields:

$$\mathcal{M}(p_1, ..., p_n; \mu^2) = \mathcal{M}_{\rm 1loop}(p_1, ..., p_n)\exp\left(-\frac{1}{2}\sum_{l=1}^{n_f} W^{\rm ew}_l(s, \mu^2)\right) \tag{52}$$

where $W^{\rm ew}_l(s, \mu^2)$ is given in Eq. (38) and the quantum numbers are those of the external fermion lines. Since at high energies all fermions can be considered massless we can again absorb the chiral top-Yukawa corrections into universal splitting functions as in Ref. [9]. Thus in the electroweak theory we find to next to leading order the corresponding probability for the emission of gauge bosons from chiral fermions subject to the cutoff μ:

$$\begin{aligned}
W^f_i(s, \mu^2) &= \frac{g^2(s)}{16\pi^2}\left[\left(T_i(T_i+1) + \tan^2\theta_{\rm w}\frac{Y_i^2}{4}\right)\left(\log^2\frac{s}{\mu^2} - 3\log\frac{s}{\mu^2}\right)\right. \\
&\quad \left. + \left(\frac{1+\delta_{f,{\rm R}}}{4}\frac{m_f^2}{M^2} + \delta_{f,{\rm L}}\frac{m_{f'}^2}{4M^2}\right)\log\frac{s}{\mu^2}\right]
\end{aligned} \tag{53}$$

The second line only contributes for left handed bottom and for top quarks as mentioned above and f' denotes the corresponding isospin partner for left handed fermions.

6 Semi-inclusive cross sections

Up to this point we have only considered the corrections from virtual corrections above the weak scale M. The physical photon, however, is massless and must be included in a semi-inclusive or fully inclusive way. It is thus necessary to consider now the regime for $\boldsymbol{k}_\perp^2 < M^2$. The corrections for external fermion, photon and $W^\pm$ lines are given in Ref. [9], in each case corresponding to the logarithmic probability to emit soft and/or collinear particles below the scale M. The high energy solution is then the boundary condition for the infrared evolution equation at the scale $\mu = M$. For the longitudinal particles, we only have corrections from the charged gauge bosons below the scale M. In this regime we also need to consider particle masses. For real photon emission we assume that the detector resolution is bounded by $\mu_{\rm expt} < M$, so that emission from real massive gauge bosons does not need to be considered and for simplicity, we restrict ourselves here to the soft photon approximation.

Under these circumstances we are now able to summarize the complete expression for observable electroweak cross sections at high energies for all universal leading and subleading Sudakov corrections as follows[6]:

$$
\begin{aligned}
& d\sigma(p_1,\ldots,p_n,g,g',\mu_{exp}) = d\sigma_{\rm Born}(p_1,\ldots,p_n,g(s),g'(s)) \\
& \times \exp\left\{-\sum_{i=1}^{n_g} W_i^g(s,M^2) - \sum_{i=1}^{n_f} W_i^f(s,M^2) - \sum_{i=1}^{n_\phi} W_i^\phi(s,M^2)\right\} \\
& \times \exp\left[-\sum_{i=1}^{n_f}\left(w_i^f(s,\mu^2) - w_i^f(s,M^2)\right) - \sum_{i=1}^{n_w}\left(w_i^{\rm w}(s,\mu^2) - w_i^{\rm w}(s,M^2)\right)\right. \\
& \qquad \left. - \sum_{i=1}^{n_\gamma} w_i^\gamma(M^2,m_j^2)\right] \times \exp\left(w_{\rm expt}^\gamma(s,m_i,\mu,\mu_{\rm expt})\right)
\end{aligned} \tag{54}
$$

The functions $W_i^\phi(s,M^2)$ and $W_i^f(s,M^2)$ are given in Eqs. (23) and (53) respectively. The remaining logarithmic probabilities are given in Ref. [9] and are summarized for convenience below:

$$
\begin{aligned}
W_i^g(s,M^2) &= \left(\frac{\alpha(s)}{4\pi}T_i(T_i+1) + \frac{\alpha'(s)}{4\pi}\left(\frac{Y_i}{2}\right)^2\right)\log^2\frac{s}{M^2} \\
&\quad - \left(\delta_{i,\rm W}\frac{\alpha(s)}{\pi}\beta_0 + \delta_{i,\rm B}\frac{\alpha'(s)}{\pi}\beta_0'\right)\log\frac{s}{M^2}
\end{aligned} \tag{55}
$$

with

$$
\beta_0 = \frac{11}{12}C_A - \frac{1}{3}n_{gen} - \frac{1}{24}n_h \quad , \quad \beta_0' = -\frac{5}{9}n_{gen} - \frac{1}{24}n_h \tag{56}
$$

[6]We emphasize that for photon and Z-boson final states the mixing effects have to be included correctly as described in Ref. [9]. In particular, for transverse degrees of freedom the corrections don't factorize with respect to the physical Born amplitude but rather with respect to the amplitudes containing the fields in the broken phase. For longitudinally polarized Z-bosons, however, there is no mixing with photons and the corrections factorize with respect to the Born amplitude.

where n_{gen} denotes the number of fermion generations [34, 35] and n_h the number of Higgs doublets. Again we note that for external photon and Z-boson states we must include the mixing appropriately as discussed in Ref. [9]. For the terms entering from contributions below the weak scale we have for fermions:

$$w_i^f(s,\mu^2) = \begin{cases} \frac{e_i^2}{(4\pi)^2}\left(\log^2\frac{s}{\mu^2} - 3\log\frac{s}{\mu^2}\right) & , \quad m_i \ll \mu \\ \frac{e_i^2}{(4\pi)^2}\left[\left(\log\frac{s}{m_i^2}-1\right)2\log\frac{m_i^2}{\mu^2}\right. & \\ \qquad \left. +\log^2\frac{s}{m_i^2} - 3\log\frac{s}{m_i^2}\right] & , \quad \mu \ll m_i \end{cases} \tag{57}$$

Analogously, for external W-bosons and photons we find:

$$w_i^{\mathrm{w}}(s,\mu^2) = \frac{e_i^2}{(4\pi)^2}\left[\left(\log\frac{s}{M^2}-1\right)2\log\frac{M^2}{\mu^2} + \log^2\frac{s}{M^2}\right] \tag{58}$$

$$w_i^\gamma(M^2,\mu^2) = \begin{cases} \frac{1}{3}\sum_{j=1}^{n_f}\frac{e_j^2}{4\pi^2}N_C^j\log\frac{M^2}{\mu^2} & , \quad m_j \ll \mu \\ \frac{1}{3}\sum_{j=1}^{n_f}\frac{e_j^2}{4\pi^2}N_C^j\log\frac{M^2}{m_j^2} & , \quad \mu \ll m_j \end{cases} \tag{59}$$

for the virtual corrections and for real photon emission we have in the soft photon approximation:

$$w_{\mathrm{expt}}^\gamma(s,m_i,\mu,\mu_{\mathrm{expt}}) = \begin{cases} \sum_{i=1}^n \frac{e_i^2}{(4\pi)^2}\left[-\log^2\frac{s}{\mu_{\mathrm{expt}}^2} + \log^2\frac{s}{\mu^2} - 3\log\frac{s}{\mu^2}\right] & , m_i \ll \mu \\ \sum_{i=1}^n \frac{e_i^2}{(4\pi)^2}\left[\left(\log\frac{s}{m_i^2}-1\right)2\log\frac{m_i^2}{\mu^2} + \log^2\frac{s}{m_i^2}\right. & \\ \left. -2\log\frac{s}{\mu_{\mathrm{expt}}^2}\left(\log\frac{s}{m_i^2}-1\right)\right] & , \mu \ll m_i \end{cases} \tag{60}$$

where n is the number of external lines and the upper case applies only to fermions since for $W^\pm$ we have $\mu < M$. Note that in all contributions from the regime $\mu < M$ we have kept mass terms inside the logarithms. This approach is valid in the entire Standard Model up to terms of order $\mathcal{O}\left(\log\frac{m_t}{M}\right)$.

7 Comparison with one loop results

In this section we compare our results from the infrared evolution equation method with the explicit one loop calculation of Ref. [36] for longitudinal $W_{\mathrm{L}}^\pm$ scattering in e^+e^- collisions and the general one loop results from Ref. [10]. In Ref. [10] all mass singular terms were isolated and the physical basis was used to obtain the DL and SL corrections from collinear terms, wave function renormalization and RG contributions. The results presented there for fermions (up to Yukawa terms) and transverse degrees

of freedom agree with our corresponding results in Ref. [9]. Also all terms calculated here are, at one loop, in agreement with Ref. [10]. The results of Ref. [36] were obtained in terms of the physical fields. We already checked that our method gives the correct terms at one loop for transverse degrees of freedom and for fermions (up to top-Yukawa terms) in Ref. [9]. Soft real photon radiation will be included in the comparison. This comparison is crucial as mentioned in section 2.1 since we must check that the splitting function approach, in particular the factorization of the DL and SL terms takes place with the same electroweak group factor $\left(\frac{g^2}{8\pi^2}T_\phi(T_\phi+1)+\frac{g'^2}{8\pi^2}\frac{Y_\phi^2}{4}\right)$ from the high effective scalar theory. Only the Yukawa terms factorize differently, namely with $\frac{g^2}{8\pi^2}$. In the following, the lower index on the cross section indicates the helicity of the electron, where e^-_- denotes the left handed electron. We summarize the relevant results for $e^+_+e^-_- \longrightarrow W^+_{\rm L}W^-_{\rm L}$ and $e^+_-e^-_+ \longrightarrow W^+_{\rm L}W^-_{\rm L}$ from Ref. [36] for convenience as follows:

$$\begin{aligned}\left(\frac{d\sigma}{d\Omega}\right)_{-,{\rm L}} \approx \left(\frac{d\sigma}{d\Omega}\right)^{\rm Born}_{-,{\rm L}}\Bigg\{1+\frac{e^2}{8\pi^2}\Bigg[&-\frac{1-2c_{\rm w}^2+4c_{\rm w}^4}{2c_{\rm w}^2s_{\rm w}^2}\log^2\frac{s}{M^2}+\frac{103-158c_{\rm w}^2+80c_{\rm w}^4}{12c_{\rm w}^2s_{\rm w}^2}\log\frac{s}{M^2}\\ &-\frac{3m_t^2}{2s_{\rm w}^2M^2}\log\frac{s}{M^2}+3\log\frac{s}{m_e^2}+2\log\frac{4\Delta E^2}{s}\left(\log\frac{s}{m_e^2}+\log\frac{s}{M^2}-2\right)\\ &-\frac{4}{3}\sum_{j=1}^{n_f}Q_j^2N_C^j\log\frac{m_j^2}{M^2}\Bigg]\Bigg\}\end{aligned} \tag{61}$$

$$\begin{aligned}\left(\frac{d\sigma}{d\Omega}\right)_{+,{\rm L}} \approx \left(\frac{d\sigma}{d\Omega}\right)^{\rm Born}_{+,{\rm L}}\Bigg\{1+\frac{e^2}{8\pi^2}\Bigg[&-\frac{5-10c_{\rm w}^2+8c_{\rm w}^4}{4c_{\rm w}^2s_{\rm w}^2}\log^2\frac{s}{M^2}+\frac{65-65c_{\rm w}^2+18c_{\rm w}^4}{6c_{\rm w}^2s_{\rm w}^2}\log\frac{s}{M^2}\\ &-\frac{3m_t^2}{2s_{\rm w}^2M^2}\log\frac{s}{M^2}+3\log\frac{s}{m_e^2}+2\log\frac{4\Delta E^2}{s}\left(\log\frac{s}{m_e^2}+\log\frac{s}{M^2}-2\right)\\ &-\frac{4}{3}\sum_{j=1}^{n_f}Q_j^2N_C^j\log\frac{m_j^2}{M^2}\Bigg]\Bigg\}\end{aligned} \tag{62}$$

The Born cross sections are given by:

$$\left(\frac{d\sigma}{d\Omega}\right)^{\rm Born}_{-,{\rm L}} = \frac{e^4}{64\pi^2 s}\frac{1}{16s_{\rm w}^4c_{\rm w}^4}\sin^2\theta \tag{63}$$

$$\left(\frac{d\sigma}{d\Omega}\right)^{\rm Born}_{+,{\rm L}} = \frac{e^4}{64\pi^2 s}\frac{1}{4c_{\rm w}^4}\sin^2\theta \tag{64}$$

These expressions demonstrate that the longitudinal cross sections in Eqs. (63) and (64) are not mass suppressed. Eqs. (61) and (62) were of course calculated in terms of the physical fields of the broken theory and in the on-shell scheme. We denote $c_{\rm w}=\cos\theta_{\rm w}$ and $s_{\rm w}=\sin\theta_{\rm w}$ respectively. Using $e=\frac{gg'}{\sqrt{g^2+g'^2}}$, $s_{\rm w}=\frac{g'}{\sqrt{g^2+g'^2}}$ and

$c_{\rm w} = \frac{g}{\sqrt{g^2+g'^2}}$ we see that the Born cross section in Eq. (63) is proportional to $(g^2+g'^2)^2$ and Eq. (63) proportional to g'^4. Below the scale where non-Abelian effects enter, we have running coupling corrections only from QED, i.e. $g^2(M^2) = \frac{e^2_{\rm eff}(M^2)}{s^2_{\rm w}}$ and $g'^2(M^2) = \frac{e^2_{\rm eff}(M^2)}{c^2_{\rm w}}$ where

$$e^2_{\rm eff}(M^2) = e^2 \left(1 + \frac{1}{3}\frac{e^2}{4\pi^2} \sum_{j=1}^{n_f} Q_j^2 N_C^j \log \frac{M^2}{m_j^2} \right) \tag{65}$$

Thus, the RG-corrections to both cross sections for $\overline{\mu} > M$ are given by (using C_A=2, $n_{gen} = 3$ and $n_h = 1$ in Eqs. 56):

$$\left(\frac{d\sigma}{d\Omega}\right)^{\rm RG}_{-,\rm L} = \left(\frac{d\sigma}{d\Omega}\right)^{\rm Born}_{-,\rm L} \left\{ 1 + \frac{e^2}{8\pi^2} \frac{41 - 82 c^2_{\rm w} + 22 c^4_{\rm w}}{6 s^2_{\rm w} c^2_{\rm w}} \log \frac{s}{M^2} \right\} \tag{66}$$

$$\left(\frac{d\sigma}{d\Omega}\right)^{\rm RG}_{+,\rm L} = \left(\frac{d\sigma}{d\Omega}\right)^{\rm Born}_{+,\rm L} \left\{ 1 + \frac{e^2}{8\pi^2} \frac{41}{6 c^2_{\rm w}} \log \frac{s}{M^2} \right\} \tag{67}$$

The Sudakov corrections to both cross sections from the infrared evolution equation method according to Eq. (54) in the soft photon approximation are given below. The quantum numbers are those of the particle-indices and are summarized in Tab. 1:

$$\begin{aligned}
\left(\frac{d\sigma}{d\Omega}\right)_{-,\rm L} = & \left(\frac{d\sigma}{d\Omega}\right)^{\rm Born}_{-,\rm L} \Bigg\{ 1 - \left(\frac{g^2}{8\pi^2} T_\phi (T_\phi + 1) + \frac{g'^2}{8\pi^2} \frac{Y^2_\phi}{4} \right) \left(\log^2 \frac{s}{M^2} - 4 \log \frac{s}{M^2} \right) \\
& - \left(\frac{g^2}{8\pi^2} T_{e^-_-} (T_{e^-_-} + 1) + \frac{g'^2}{8\pi^2} \frac{Y^2_{e^-_-}}{4} \right) \left(\log^2 \frac{s}{M^2} - 3 \log \frac{s}{M^2} \right) \\
& -3 \frac{g^2}{16\pi^2} \frac{m_t^2}{M^2} \log \frac{s}{M^2} - \frac{e^2}{8\pi^2} \Bigg[\left(\log \frac{s}{m_e^2} - 1 \right) 2 \log \frac{m_e^2}{\mu^2} \\
& + \log^2 \frac{s}{m_e^2} - 3 \log \frac{s}{m_e^2} - \log^2 \frac{s}{M^2} + 3 \log \frac{s}{M^2} \\
& +2 \left(\log \frac{s}{M^2} - 1 \right) \log \frac{M^2}{\mu^2} - \left(\log \frac{s}{m_e^2} - 1 \right) \left(2 \log \frac{m_e^2}{\mu^2} - 2 \log \frac{s}{\mu^2_{exp}} \right) - \\
& 2 \left(\log \frac{s}{M^2} - 1 \right) \left(\log \frac{M^2}{\mu^2} - \log \frac{s}{\mu^2_{exp}} \right) - \log^2 \frac{s}{m_e^2} - \log^2 \frac{s}{M^2} \Bigg] \\
& + \frac{2}{3} \frac{e^2}{4\pi^2} \sum_{j=1}^{n_f} Q_j^2 N_C^j \log \frac{M^2}{m_j^2} \Bigg\} \\
= & \left(\frac{d\sigma}{d\Omega}\right)^{\rm Born}_{-,\rm L} \Bigg\{ 1 - \frac{e^2}{8\pi^2} \left(\frac{1 + 2 c^2_{\rm w}}{2 s^2_{\rm w} c^2_{\rm w}} \log^2 \frac{s}{M^2} - 7 \frac{1 + 2 c^2_{\rm w}}{4 s^2_{\rm w} c^2_{\rm w}} \log \frac{s}{M^2} \right) + \frac{e^2}{8\pi^2} \times
\end{aligned}$$

	T	Y	Q
e^-_-	1/2	-1	-1
e^-_+	0	-2	-1
e^+_+	1/2	1	1
e^+_-	0	2	1
u_-	1/2	1/3	2/3
u_+	0	4/3	2/3
d_-	1/2	1/3	-1/3
d_+	0	-2/3	-1/3
$W^\pm$	1	0	±1
$\phi^\pm$	1/2	±1	±1
χ	1/2	-1	0
H	1/2	$+1$	0

Table 1: The quantum numbers of various particles in the electroweak theory. The indices indicate the helicity of the electrons and quarks. We neglect all mass terms, i.e. consider all particles as chiral eigenstates with well defined total weak isospin (T) and weak hypercharge (Y) quantum numbers. In each case, the electric charge Q, measured in units of the proton charge, by the Gell-Mann-Nishijima formula $Q = T^3 + Y/2$. For longitudinally polarized gauge bosons, the associated scalar Goldstone bosons describe the DL asymptotics.

$$\left[2\log^2\frac{s}{M^2}-3\log\frac{m_e^2}{M^2}-4\log\frac{s}{\mu_{exp}^2}\left(\log\frac{s}{m_eM}-1\right)-\frac{3m_t^2}{2s_{\rm w}^2M^2}\log\frac{s}{M^2}\right]$$
$$+\frac{2}{3}\frac{e^2}{4\pi^2}\sum_{j=1}^{n_f}Q_j^2N_C^j\log\frac{M^2}{m_j^2}\Bigg\} \tag{68}$$

Adding Eqs. (66) and (68) yields exactly the result in Eq. (61) from Ref. [36]. Analogously, we have for right handed electrons:

$$\begin{aligned}
\left(\frac{d\sigma}{d\Omega}\right)_{+,\mathrm{L}} &= \left(\frac{d\sigma}{d\Omega}\right)^{\rm Born}_{+,\mathrm{L}}\Bigg\{1-\left(\frac{g^2}{8\pi^2}T_\phi(T_\phi+1)+\frac{g'^2}{8\pi^2}\frac{Y_\phi^2}{4}\right)\left(\log^2\frac{s}{M^2}-4\log\frac{s}{M^2}\right)\\
&-\left(\frac{g^2}{8\pi^2}T_{e^-_+}(T_{e^-_+}+1)+\frac{g'^2}{8\pi^2}\frac{Y^2_{e^-_+}}{4}\right)\left(\log^2\frac{s}{M^2}-3\log\frac{s}{M^2}\right)\\
&-3\frac{g^2}{16\pi^2}\frac{m_t^2}{M^2}\log\frac{s}{M^2}-\frac{e^2}{8\pi^2}\Bigg[\left(\log\frac{s}{m_e^2}-1\right)2\log\frac{m_e^2}{\mu^2}\\
&+\log^2\frac{s}{m_e^2}-3\log\frac{s}{m_e^2}-\log^2\frac{s}{M^2}+3\log\frac{s}{M^2}\\
&+2\left(\log\frac{s}{M^2}-1\right)\log\frac{M^2}{\mu^2}-\left(\log\frac{s}{m_e^2}-1\right)\left(2\log\frac{m_e^2}{\mu^2}-2\log\frac{s}{\mu_{exp}^2}\right)-\\
&2\left(\log\frac{s}{M^2}-1\right)\left(\log\frac{M^2}{\mu^2}-\log\frac{s}{\mu^2_{exp}}\right)-\log^2\frac{s}{m_e^2}-\log^2\frac{s}{M^2}\Bigg]\\
&+\frac{2}{3}\frac{e^2}{4\pi^2}\sum_{j=1}^{n_f}Q_j^2N_C^j\log\frac{M^2}{m_j^2}\Bigg\}\\
&= \left(\frac{d\sigma}{d\Omega}\right)^{\rm Born}_{+,\mathrm{L}}\Bigg\{1-\frac{e^2}{8\pi^2}\left(\frac{5-2c_{\rm w}^2}{4s_{\rm w}^2c_{\rm w}^2}\log^2\frac{s}{M^2}-\frac{4-c_{\rm w}^2}{s_{\rm w}^2c_{\rm w}^2}\log\frac{s}{M^2}\right)+\frac{e^2}{8\pi^2}\times\\
&\left[2\log^2\frac{s}{M^2}-3\log\frac{m_e^2}{M^2}-4\log\frac{s}{\mu_{exp}^2}\left(\log\frac{s}{m_eM}-1\right)-\frac{3m_t^2}{2s_{\rm w}^2M^2}\log\frac{s}{M^2}\right]\\
&+\frac{2}{3}\frac{e^2}{4\pi^2}\sum_{j=1}^{n_f}Q_j^2N_C^j\log\frac{M^2}{m_j^2}\Bigg\}
\end{aligned} \tag{69}$$

Again we see that after adding Eqs. (67) and (69) we obtain the result in Eq. (62) from Ref. [36]. Thus we have demonstrated that to subleading logarithmic accuracy our results from the infrared evolution equation method in conjunction with the Goldstone boson equivalence theorem are identical with existing one loop calculations with physical fields in the high energy limit.

8 Discussion of the results

In this section we discuss the size of the subleading Sudakov corrections obtained in this work. We neglect renormalization group corrections for simplicity and use $\frac{e^2}{4\pi} = \frac{1}{137}$, $\frac{g^2}{4\pi} = \frac{e^2(M^2)}{s_{\rm w}^2 4\pi} = \frac{1}{0.23\times 128}$ and $\frac{g'^2}{4\pi} = \frac{e^2(M^2)}{c_{\rm w}^2 4\pi} = \frac{1}{0.77\times 128}$. The motivation for investigating the size of the gauge invariant corrections at the subleading level is two-fold. While this discussion is incomplete for processes with a large angular dependence, it is nevertheless useful in estimating how good the DL approximation is at higher orders. In addition, we gain physical insight into the importance of Yukawa corrections and the partial cancellation between subleading terms.

8.1 Sudakov effects for longitudinal gauge boson and Higgs production

We begin with the Yukawa corrections for external scalars given in Eq. (23) with the infrared cutoff $\mu = M$. Using the quantum numbers of Tab. 1, we have

$$
\begin{aligned}
-W_i^\phi(s,M^2) &= -\frac{g^2}{16\pi^2}\left[\left(\frac{1}{2}\left(\frac{1}{2}+1\right)+\tan^2\theta_{\rm w}\frac{1}{4}\right)\left(\log^2\frac{s}{M^2}-4\log\frac{s}{M^2}\right)+\frac{3}{2}\frac{m_t^2}{M^2}\log\frac{s}{M^2}\right] \\
&= -\frac{g^2}{16\pi^2}\left[0.79\log^2\frac{s}{M^2}+4.01\log\frac{s}{M^2}\right]
\end{aligned}
\tag{70}
$$

where we use $M = 80$ GeV, $m_t = 175$ GeV and $s_{\rm w}^2 = 0.23$. The first thing to notice is that the Yukawa enhanced logarithms dominate over the subleading Sudakov corrections and enhance the overall Sudakov suppression. At 1 TeV we have $\log\frac{s}{M^2} = 5.05$ and thus almost equal contributions from DL and SL terms. At 2 TeV we have $\log\frac{s}{M^2} = 6.44$ and at 3 TeV $\log\frac{s}{M^2} = 7.25$. In real calculations, however, one finds that the Yukawa terms are always proportional to $\log\frac{s}{m_t^2}$. Since the factor of the Yukawa logarithm is uniquely determined by Eq. (23) we can replace the respective mass term inside the logarithm[7] Thus, for $\phi^\pm$ for instance, we have to consider

$$
\begin{aligned}
-W_i^\phi(s,M^2) &= -\frac{g^2}{16\pi^2}\left[\left(\frac{1}{2}\left(\frac{1}{2}+1\right)+\tan^2\theta_{\rm w}\frac{1}{4}\right)\left(\log^2\frac{s}{M^2}-4\log\frac{s}{M^2}\right)+\frac{3}{2}\frac{m_t^2}{M^2}\log\frac{s}{m_t^2}\right] \\
&\approx -\frac{g^2}{16\pi^2}\left[0.79\log^2\frac{s}{M^2}+4.01\log\frac{s}{M^2}-11.24\right]
\end{aligned}
\tag{71}
$$

With the above mass values we have at one loop about 40 (39) % at 1 (3) TeV from the subleading terms relative to the DL corrections and at the two loop level about 79 (77) % at 1 (3) TeV. The subleading corrections are therefore non-negligible and

[7] Analogously for χ we can put $M = M_{\rm Z}$ and for H we have $M = M_{\rm H}$ as arguments of the non-Yukawa logarithms in Eq. (23) depending on which mass is the largest in a given process.

enhancing the Sudakov suppression. Even at 100 TeV the subleading terms make up about 52 % (!) at the two loop level and must be taken into account. The good news is that the absolute size of the DL correction per line according to Eq. (71) at the one loop level is 5.7 (11.7) % at 1 (3) TeV and at two loops 0.16 (0.7) % at 1 (3) TeV relative to the Born cross section.

The above numbers are valid for both external H and χ fields. For $\phi^\pm$ we also have to consider the purely electromagnetic corrections according to Eq. (54). Thus we have on the level of the cross section for each longitudinally polarized W-boson including soft photon radiation:

$$-w_i^{\mathrm{w}}(s,\mu^2) + w_i^{\mathrm{w}}(s,M^2) + w^{\gamma}_{\mathrm{expt}}(s,M,\mu,\mu_{\mathrm{expt}}) = \frac{e^2}{16\pi^2}\left[\log^2\frac{s}{M^2} - 2\log\frac{s}{\mu^2_{\mathrm{expt}}}\left(\log\frac{s}{M^2}-1\right)\right] \tag{72}$$

Thus, the complete size of the corrections for $\phi^\pm$ on the level of the cross section, chosing $\mu_{\mathrm{expt}} = M$, is given by

$$-W_i^{\phi}(s,M^2) - w_i^{\mathrm{w}}(s,\mu^2) + w_i^{\mathrm{w}}(s,M^2) + w^{\gamma}_{\mathrm{expt}}(s,M,\mu,M) = -\frac{g^2}{16\pi^2}\left[1.02\log^2\frac{s}{M^2} + 2.01\log\frac{s}{M^2} - 11.24\right] \tag{73}$$

It is clear that the DL approximation is much more appropriate for longitudinal W-bosons than for the neutral external scalars. For instance we have about 6.5 (4.3) % at 1 (3) TeV from the subleading terms relative to the DL contributions. The absolute size of the DL corrections relative to the Born cross section at one loop is 7 (15) % at 1 (3) TeV and at two loops 0.26 (1.1) % at 1 (3) TeV. The subleading terms at the two loop level contribute about 13 (8.6) % relative to the DL corrections.

8.2 Sudakov effects for quarks of the third generation

In order to estimate the size of the corrections for chiral heavy quark production we consider first the case of left handed bottom and top quarks. In this case we have from Tab. 1:

$$\begin{aligned} -W_i^{t_{\mathrm{L}},b_{\mathrm{L}}}(s,M^2) &= -\frac{g^2}{16\pi^2}\left[\left(\frac{1}{2}\left(\frac{1}{2}+1\right)+\tan^2\theta_{\mathrm{w}}\frac{1}{36}\right)\left(\log^2\frac{s}{M^2} - 3\log\frac{s}{M^2}\right) + \frac{m_t^2}{4M^2}\log\frac{s}{m_t^2}\right] \\ &\approx -\frac{g^2}{16\pi^2}\left[0.814\log^2\frac{s}{M^2} - 1.246\log\frac{s}{M^2} - 1.87\right] \end{aligned} \tag{74}$$

The corrections of purely electromagnetic origin are different for the two cases. In general we have

$$-w_i^{f}(s,\mu^2) + w_i^{f}(s,M^2) + w^{\gamma}_{\mathrm{expt}}(s,m_f,\mu,M) = \frac{e^2Q_f^2}{16\pi^2}\left[-\log^2\frac{s}{M^2} - \log\frac{M^2}{m_f^2}\left(2\log\frac{s}{M^2}-3\right) + 2\log\frac{s}{M^2}\right] \tag{75}$$

The full result for the left handed top quark is therefore given by:

$$-W_i^{t_{\rm L}}(s,M^2) - w_i^{t_{\rm L}}(s,\mu^2) + w_i^{t_{\rm L}}(s,M^2) + w^{\gamma}_{\rm expt}(s,m_t,\mu,M) =$$
$$-\frac{g^2}{16\pi^2}\left[0.916\log^2\frac{s}{M^2} - 1.769\log\frac{s}{M^2} - 1.39\right] \tag{76}$$

Thus we see that there is a partial cancellation between the subleading and the Yukawa terms and the overall DL suppression is somewhat reduced. In relative terms at one loop, the SL corrections are about 33 (20) % at 1 (3) TeV, and at two loops the relative size of the subleading terms is 65 (41) % at 1 (3) TeV. The absolute size of the DL corrections at one loop is 5.4 (11.1) % per line at 1 (3) TeV. At two loops we have corrections of 0.15 (0.62) % at 1 (3) TeV relative to the Born cross section.

The full result for the left handed bottom quark (with $m_b = 4.5$ GeV) is given by:

$$-W_i^{b_{\rm L}}(s,M^2) - w_i^{b_{\rm L}}(s,\mu^2) + w_i^{b_{\rm L}}(s,M^2) + w^{\gamma}_{\rm expt}(s,m_b,\mu,M) =$$
$$-\frac{g^2}{16\pi^2}\left[0.829\log^2\frac{s}{M^2} - 1.002\log\frac{s}{M^2} - 2.31\right] \tag{77}$$

The partial cancellation between the subleading and the Yukawa terms and the overall DL suppression is reduced. In relative terms at one loop, the SL corrections are about 43 (29) % at 1 (3) TeV, and at two loops the relative size of the subleading terms is 86 (58) % at 1 (3) TeV. The absolute size of the DL corrections at one loop is 5.9 (12.2) % per line at 1 (3) TeV. At two loops we have corrections of 0.18 (0.75) % at 1 (3) TeV relative to the Born cross section.

For a right handed top quark we have from Tab. 1:

$$\begin{aligned} -W_i^{t_{\rm R}}(s,M^2) &= -\frac{g^2}{16\pi^2}\left[\frac{4}{9}\frac{s_{\rm w}^2}{c_{\rm w}^2}\left(\log^2\frac{s}{M^2} - 3\log\frac{s}{M^2}\right) + \frac{m_t^2}{2M^2}\log\frac{s}{m_t^2}\right] \\ &\approx -\frac{g^2}{16\pi^2}\left[0.133\log^2\frac{s}{M^2} + 2.26\log\frac{s}{M^2} - 3.746\right] \end{aligned} \tag{78}$$

Now we need to add the corrections from Eq. (75). The full result is thus

$$-W_i^{t_{\rm R}}(s,M^2) - w_i^{t_{\rm R}}(s,\mu^2) + w_i^{t_{\rm R}}(s,M^2) + w^{\gamma}_{\rm expt}(s,m_t,\mu,M) =$$
$$-\frac{g^2}{16\pi^2}\left[0.235\log^2\frac{s}{M^2} + 1.737\log\frac{s}{M^2} - 3.27\right] \tag{79}$$

Thus we see that there is a large correction of the top Yukawa terms (a factor of 7.4 for the relative coefficients) and the overall DL suppression is strongly enhanced. In relation to the DL contribution at one loop, the SL corrections are about 69 (60) % at 1 (3) TeV, and at two loops the relative size of the subleading terms is 139 (120) % at 1 (3) TeV. The absolute size of the DL corrections at one loop is 1.6 (3.3) % per line at 1 (3) TeV. At two loops, however, we have corrections of only 0.013 (0.055) % at 1

(3) TeV relative to the Born cross section. Thus, the apparent lack of convergence of the DL approximation is irrelevant for practical purposes.

From a physical point of view it is clear how the large subleading terms can be understood. Right handed fermions in general couple only to photons and Z-bosons and the coupling to the Z-boson is proportional to $\left(\frac{\alpha'}{4\pi}\right)$ for the DL and non-Yukawa SL corrections. The Yukawa corrections, however are proportional to $\left(\frac{\alpha}{4\pi}\right)$ and in addition for the right handed top, we have:

$$\begin{aligned} \frac{m_t^2}{2M^2} &\sim 5.38\frac{Y_t^2}{4} = 5.38\frac{4}{9} &(80)\\ \alpha(M^2) &\sim 3.35\alpha'(M^2) &(81)\end{aligned}$$

The effect is somewhat softened by the electromagnetic corrections from Eq. (75). In general the size of the subleading terms cannot be neglected at the two loop level for all Yukawa enhanced Sudakov corrections discussed in this work.

9 Conclusions

In this paper we calculated the universal subleading $\mathcal{O}\left(g^{2n}\log^{2n-1}\frac{s}{M^2}, g'^{2n}\log^{2n-1}\frac{s}{M^2}\right)$ logarithmic Sudakov corrections to longitudinal gauge boson and Higgs production to all orders. We have employed the infrared evolution equation method and used the equivalence theorem to obtain the high energy kernel of the equation for longitudinal gauge boson production. All Yukawa enhanced SL Sudakov terms in non-mass suppressed amplitudes are universal to all orders. This feature is evident in the splitting function formalism which we have adopted to calculate the virtual Sudakov corrections. The approach, concerning in particular the novel Yukawa enhanced subleading corrections, has been verified by employing a non-Abelian generalization of Gribov's bremsstrahlung theorem. We agree with the literature at the one loop level, which is a highly non-trivial check considering the complicated nature of electroweak radiative corrections and can serve as an independent confirmation of those results. In addition this comparison confirms the validity of the splitting function approach since DL and non-Yukawa SL corrections factorize with respect to the same group factor of the effective high energy theory. These SL contributions are determined by the spin only and are thus identical to those found in a scalar theory with an unbroken $SU(2) \times U(1)$, while the Yukawa enhanced SL corrections indicate the spontaneously broken gauge symmetry.

The physical picture which is now emerging is clear: at high energies the SM behaves like an unbroken gauge theory up to DL and SL accuracy for fermions and transversely polarized gauge bosons. Only Yukawa corrections are novel features in this picture. For longitudinally polarized gauge bosons and Higgs scalars, the effective theory is given by the Goldstone boson equivalence theorem and contains corrections

in analogy to a non-Abelian gauge theory with scalar fields in the fundamental representation. Again, Yukawa terms modify this picture as a unique ingredient of broken gauge theories. The mass gap between the electroweak gauge bosons can be included in a natural way via the matching conditions in the framework of the infrared evolution equation method. Thus all universal Sudakov corrections to DL and SL accuracy are known in the electroweak theory to all orders. The remaining corrections which enter at this level of precision are given by angular terms of the type $\log \frac{u}{t} \log \frac{s}{M^2}$. These terms are non-universal and don't factorize with respect to the Born amplitude. While these terms are known at one loop, for phenomenological applications at future colliders a two loop analysis is desirable. In addition, subleading RG effects of the type $\alpha^n \beta_0 \log^{2n-1} \frac{s}{M^2}$ coupling effects at higher order should be consistently resummed via the inclusion of a running coupling in each loop analogously to the QCD Sudakov form factor.

In summary, all universal Sudakov logarithms in the electroweak SM are known at the subleading level to all orders and are non-negligible at future collider energies. The inclusion of the leading and full subleading electroweak radiative corrections at least at the two loop level will be important in investigating new physics effects at TeV energies.

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